Algebra I Strand

The Algebra I strand builds upon students' knowledge and skills developed from their experience with linear functions, tables, graphs, verbal rules, and symbolic rules. As students deepen their understanding of relations and functions, they will expand their capacity to make meaningful use of new types of functions, including polynomial and exponential functions. Important new learning includes combining functions, expressing functions in equivalent forms, and finding inverses. This experience leads to more global understanding of classes of functions as a concept and the recognition of the significant characteristics of various classes.

To demonstrate achievement in the Algebra I strand, students must also develop insights into mathematical abstraction and structure. Students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions. As students become more familiar with these types of abstractions, they develop the means to solve equations and inequalities, express equivalent forms, and assert proofs.

Facility with abstraction and deeper knowledge of functions and relations give students more powerful mathematical tools to analyze and describe situations. Tools such as graphs and other visual representations of phenomena provide additional insights into problems and applications.

Standards in the Algebra I strand include performing operations such as opposite (additive inverse), reciprocal, and root; solving equations and inequalities with absolute values; simplifying expressions; solving multi-step problems with linear equations and inequalities; graphing linear equations and finding the *x*- and *y*-intercepts; verifying points on a line given an equation; deriving linear equations; understanding and using the relationship between parallel lines and slopes; solving systems of linear equations, including meaningfully interpreting their graphical representations; performing operations and solving multi-step problems with monomials and polynomials; and solving rate, work, and percent mixture problems.

The ten specific California academic content standards covered by the CAHSEE Algebra I strand are discussed in the following pages.

Strand	Algebra I (1A)	If $x = -7$, then $-x =$	
Standard 1A2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, and taking a root, and raising to a fractional power. They understand and use the rules of		A -7 B $-\frac{1}{7}$ C $\frac{1}{7}$	
exponents.*	ind use the rules of	D 7	M02863
Construct	Conceptual Understanding		

Facility with inverse operations is critical for students as they solve equations and inequalities. CAHSEE test questions for this content standard focus on two of the stated components: finding the opposite (additive inverse) and finding the reciprocal. As students gain facility in algebraic reasoning, they should recognize the usefulness of finding the additive inverse and reciprocal in simplifying equations and inequalities.

Sample Test Question

The sample test question gives a value for x and asks for the opposite. The correct answer is choice D. Students should change the sign of -7 to 7.

Analysis of Distractors

The distractors represent misunderstanding of taking the opposite. Distractor A represents neglecting to change the sign. Distractor B represents taking the reciprocal. Distractor C represents taking the reciprocal and the opposite.

^{*} The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand	Algebra I (1A)	If x is an integer, what is the solution to $ x-3 < 1$?	
	1A3.0 re equations and nvolving absolute	A $\{-3\}$ B $\{-3, -2, -1, 0, 1\}$ C $\{3\}$	
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	C {3} D {-1, 0, 1, 2, 3}	M03035

For success in algebra, students should understand the concept of absolute value and the ways its meaning is used in solving equations and inequalities. CAHSEE test questions for this standard focus on both of its components: solving equations involving absolute values and solving inequalities involving absolute values. On the examination, the tested inequalities will involve only integers.

Sample Test Question

In the sample question, students are asked to solve an inequality involving absolute value. The correct answer is choice C. Students should recognize that this inequality, |x-3| < 1, is equivalent to -1 < x - 3 < 1. By adding 3 to each member of the inequality, the following equivalence is obtained: 2 < x < 4. Since x is an integer, the solution is 3 because 3 is the only integer greater than 2 and less than 4.

Analysis of Distractors

Distractor A is the negative of the correct answer and results from a misunderstanding of absolute value. Distractors B and D result from setting up the inequality improperly and then adding –3 to only two members of the inequality.

Standard 1A4.0

Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5)+4(x-2)=12.

Constructs Conceptual

Understanding, Problem Solving Which of the following is equivalent to 4(x+5)-3(x+2)=14?

A
$$4x + 20 - 3x - 6 = 14$$

B
$$4x+5-3x+6=14$$

C
$$4x+5-3x+2=14$$

D
$$4x + 20 - 3x - 2 = 14$$

M02936

Finding ways to simplify expressions before trying to solve an equation or inequality is a valuable problem-solving skill. CAHSEE test questions for this content standard involve both equations and inequalities and focus on simplification rather than solution.

Sample Test Question

In the sample question, students are asked to identify an equation that is equivalent to the given equation, thereby determining the first step in simplification. The correct answer is choice A. Students should understand that for the given equation the first step in simplifying is to expand the quantities in parentheses with each coefficient: 4x + 20 - 3x - 6 = 14.

Analysis of Distractors

The distractors present incorrect simplifications. Distractor B presents the failure to multiply 4 by 5 in the first set of parentheses and the error of obtaining a product of +6 from multiplying -3 by 2. Distractor C presents the failure to multiply 4 by 5 in the first set of parentheses and -3 by 2 in the second set of parentheses. Distractor D presents the failure to multiply -3 by 2 in the second set of parentheses.

Standard 1A5.0

Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Constructs Procedural

Skills, Conceptual Understanding, Problem Solving Solve for x.

$$5(2x-3)-6x<9$$

A x < -1.5

B x < 1.5

 \mathbf{C} x < 3

 $\mathbf{D} \quad x < 6$

102020

Solving multi-step problems and word problems can help students develop problem solving skills and enhance their ability to think algebraically. CAHSEE test questions for this standard focus on both of its components: finding solutions to linear equations and inequalities and providing justification for each step in the solution. Justification requires the identification of appropriate specific steps in the solution process that contribute to solving the equation or inequality.

Sample Test Question

The sample question presents an inequality and asks students to solve for the variable. The correct answer is choice D. Students should recognize that the inequality requires expanding the quantity in parentheses by multiplying it by the coefficient, combining like terms, adding 15 to both sides of the equation, and dividing both sides by 4 to obtain the value for x.

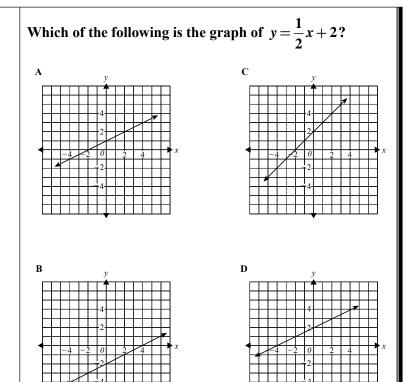
Analysis of Distractors

The distractors present mistakes in the solution process. Distractor A is obtained by subtracting 15 in the third step. Distractor B is obtained by subtracting 15 in the third step and dividing by -4 in the fourth step, or finding +6 as the sum of 15+9. Distractor C is obtained by finding 8x as the sum of 2x and -6x after failing to multiply 5 by 2 in the first set of parentheses.

Standard 1A6.0 Students graph a linear equation and compute the x-and y-intercepts (e.g., graph 2x + 6y = 4). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by 2x + 6y < 4).*

Constructs **Procedural**

Skills, Conceptual Understanding



CAHSEE test questions for this standard focus on two components: graphing the linear equation and finding the x- and y-intercepts. Students may be asked to identify the graph that corresponds to a given equation or to identify the equation that corresponds to a given graph. The x- and y-intercepts may be identified by a single number or a coordinate pair.

Sample Test Question

The sample question presents an equation and asks students to identify the correct graph of the equation from among four choices. The correct answer is choice D. Students should recognize that the equation, $y = \frac{1}{2}x + 2$, is in the form y = mx + b and use this information to determine the correct graph: The variable b represents the y-intercept, which in this case is 2, and the variable m represents the slope of the graph, which in this case is $\frac{1}{2}$.

^{*} The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Analysis of Distractors

Distractor A displays a line with a slope of $\frac{1}{2}$ and y-intercept of 1. Distractor B displays a line with a slope of $\frac{1}{2}$ and y-intercept of -2. Distractor C displays a line with a slope of 1 and y-intercept of 2.

73

Standard 1A7.0

Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations. by using the point slope formula.*

Constructs **Procedural Skills**,

Conceptual Understanding, Problem Solving Which of the following points lies on the line 4x + 5y = 20?

- **A** (0, 4)
- \mathbf{B} (0, 5)
- C (4, 5)
- **D** (5, 4)

CAHSEE test questions for this standard require students to select a set of one or more points, either by ordered pairs or by graphical location, that lie on the graph of a given linear equation, or to select an equation, either by its algebraic notation or by its graph, whose graph includes one or more specified points. Components of the standard include verifying that a point lies on a given line and deriving an equation from information given about the line. To verify that points do or do not lie on a given line, students may use substitution of *x*- or *y*- values to find corresponding ordered pairs.

Sample Test Question

The sample question presents an equation and asks students to identify an ordered pair that would lie on the graph of that equation. The correct answer is choice A. Students should recognize that the equation 4x + 5y = 20 represents true statements for certain corresponding pairs of values for x and y. The values may be tested by substituting them for x and y, respectively, in the equation. If x = 0 and y = 4, then 4(0) + 5(4) = 20 is a true statement.

Analysis of Distractors

Distractor B would mean substituting 0 for x and 5 for y, the result being that 4(0) + 5(5) = 25 rather than 20. Distractor C means substituting 4 for x and 5 for y, the result being that 4(4) + 5(5) = 41 rather than 20. Distractor D means substituting 5 for x and 4 for y, the result being that 4(5) + 5(4) = 40 rather than 20.

^{*} The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Standard 1A8.0

Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.*

Constructs Conceptual

Understanding, Problem Solving What is the slope of a line parallel to the line

$$y = \frac{1}{3}x + 2?$$

- $\mathbf{A} = -3$
- **B** $-\frac{1}{3}$
- $\mathbf{C} = \frac{1}{2}$
- **D** 2

To demonstrate understanding of this content standard, students must know that parallel lines have equivalent slopes and different *x*- and *y*-intercepts. CAHSEE test questions for this standard may require students to find the slope of a line parallel to a given line, to identify pairs of parallel lines from their slopes, or to identify lines not parallel to a given line from a given or derived slope.

Sample Test Question

The sample question asks students to determine the slope of a line parallel to a given line. The correct answer is choice C. Students should know that parallel lines have equivalent slopes. They must also recognize that the slope of the line that is represented by the equation

$$y = \frac{1}{3}x + 2$$
 is $\frac{1}{3}$.

Analysis of Distractors

Distractor A is the negative reciprocal of the slope of the given equation. It is also the slope of y = -3x + 2. Distractor B is the additive inverse of the slope of the given equation. It is also the

slope of $y = -\frac{1}{3}x + 2$. Distractor D is the *y*-intercept of the given equation.

^{*} The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Standard 1A9.0

Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Constructs Procedural Skills,

Conceptual Understanding, Problem Solving

$$\begin{cases}
7x + 3y = -8 \\
-4x - y = 6
\end{cases}$$

What is the solution to the system of equations shown above?

A
$$(-2, -2)$$

B
$$(-2, 2)$$

$$C$$
 (2, -2)

M02956

Many real-world situations are most appropriately modeled as systems of equations, and graphs of these kinds of systems are common in newspapers and other media. CAHSEE test questions for this content standard focus on four components: solving systems of linear equations, interpreting the solutions graphically, solving a system of linear inequalities, and determining the solution sets.

Sample Test Question

The sample question presents a system of equations and asks students to find the solution. The correct answer is choice B. To solve this sample problem, students should use a method such as the following:

(multiply the second equation by 3)
$$-12x-3y=18$$
(add to the first equation)
$$-5x=10$$
(divide by 5)
$$x=-2$$
(substitute $x=-2$ in the first equation)
$$7(-2)+3y=-8$$
(expand)
$$-14+3y=-8$$
(combine)
$$3y=6$$

(divide by 3)
$$y = 2$$

Analysis of Distractors

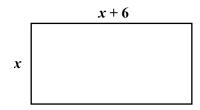
Distractor A results from errors in substitution and/or computation, as does Distractor D. Distractor C presents a misunderstanding of the correct order of the *x* and *y* values.

Standard 1A10.0

Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

Constructs Procedural Skills,

Conceptual Understanding, Problem Solving



The length of the rectangle above is 6 units longer than the width. Which expression could be used to represent the area of the rectangle?

$$\mathbf{A} \quad x^2 + 6x$$

B
$$x^2 - 36$$

C
$$x^2 + 6x + 6$$

D
$$x^2 + 12x + 36$$

M00402

This standard requires students to accurately execute arithmetic operations on monomials and polynomials and to select and use these techniques to solve problems.

Sample Test Question

The sample question presents a rectangle with length and width labeled with a monomial and a binomial, and students are asked to determine the expression that would correctly represent the area. The correct answer is choice A. Students must know to multiply the length by the width in order to find the area and then perform the computation correctly.

Analysis of Distractors

Distractors B, C, and D represent examples of incorrect multiplication of the monomial by the binomial.

Standard 1A15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

Constructs **Procedural Skills**,

Conceptual Understanding,

Problem Solving

Mr. Jacobs can correct 150 quizzes in 50 minutes. His student aide can correct 150 quizzes in 75 minutes. Working together, how many minutes will it take them to correct 150 quizzes?

A 30

B 60

C 63

D 125

M03000

The key assessment of student learning in any subject is whether or not students can apply their knowledge and skills to a new problem situation. CAHSEE test questions written for this standard focus on students' ability to apply their mathematical skills and knowledge to solve rate problems, work problems, and percent mixture problems.

Sample Test Question

The sample question presents a rate problem in the context of grading quizzes. The correct answer is choice A. Students should recognize that correcting 150 quizzes in 50 minutes is equivalent to the rate of 3 quizzes per minute and that correcting 150 quizzes in 75 minutes is equivalent to the rate of 2 quizzes per minute. Working together, Mr. Jacobs and his aide can correct 5 quizzes each minute: 150 quizzes ÷ 5 quizzes/minute = 30 minutes.

Analysis of Distractors

The distractors represent misunderstandings of a rate problem. Distractor B represents the number of minutes it would take both individuals to complete the task at the rate of 2.5 quizzes/minute—the average of their rate. Distractor C is the approximate average of their time for 150 quizzes. Distractor D is the sum of the number of minutes each takes to correct 150 quizzes (50+75).